XVIII. *On the Reſolution of attractive Powers.* *By* Edward Waring, *M. D. F. R. S. and* Lucaſian *Profeſſor of Mathematics at* Cambridge.

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1. A FORCE acting at a given point may be reſolved by an infinite number of ways into two, three, or more (n) forces acting at the ſame point, either in the ſame or different planes with the given force and each other; and, *vice verſâ*, any number of ſuch forces acting in the ſame or different planes may be reduced into one.

Ex. Fig. 1. Tab. III. Let a body A be acted on by three forces AB, AC, and AD, not being in the ſame plane; reduce any two of them AB and AC to one AE, by compleating the parallelogram ABEC; then reduce the two forces AE and AD to one AF by completing the parallelogram AEFD, and the three forces AB, AC, and AD, are reduced to the one AF.

2. If n forces act on the body A at the ſame time, and any $(n-1)$ of them be reduced to one, the force reſulting will be ſituated in the ſame plane with the remaining, and force equivalent to the (n) forces.

3. If one force a be reſolved into ſeveral others $x, y, z, v,$ &c. ſituated in different planes, and the ſines of the angles, which the forces $y, z, v,$ &c. contain with the plane made by the direction of the forces x and a be reſpectively s, s', s'', &c. then will $sy \pm s'z \pm s''v \pm$ &c. $= 0$.

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PROBLEM I.

Fig. 2. Given the law of attraction of each of the parts of a given line in terms of their diſtance from a given point P; to find the attraction of the whole line *ab* on the point P.

Find the attraction of the line *ab* on the point P in the two directions P*f* and *fb* by the following method. Draw P*x* from the point P to any point *x* of the line *ab*, the force acting on the point P by the particle *xy* will be the given function (determined from the given law of attraction) of the diſtance into the particle; draw alſo P*b* perpendicular from the point P to the line *ab*, and let P*f* = *a*, *bf* = *b*, and *fx* = *y*; then will the diſtance $Px = \sqrt{(a^2 \pm 2by + y^2)}$, and the function of the diſtance into the particle $xy = \varphi\left(\sqrt{(a^2 \pm 2by + y^2)}\right) \times \dot{y} = F(y) \times \dot{y}$; let this be denoted by *lx* ſituated in the line P*x*, which reſolve into two others $nx = \frac{y\dot{y} \times F : (y)}{Px = \sqrt{(a^2 \pm 2by + y^2)}}$ ſituated in the line *ab*, and *ln* (in a direction parallel to P*f*) $= \frac{a\dot{y} \times F : (y)}{\sqrt{(a^2 \pm 2by + y^2)}}$; find the fluents of the fluxions $\frac{y\dot{y} \times F : (y)}{Px}$ and $\frac{a\dot{y} \times F : (y)}{Px}$ contained between the values *af* and *fb* of the line *fx* = *y*, which ſuppoſe Y and V reſpectively; through the point *p* draw P*y* parallel to *fb* = Y, and in the line P*f* aſſume P*u* = V; complete the parallelogram P*uzy*; P*z* will be the force of the line *ab* on the point P.

Cor. If F : (*y*) varies as any power or root (2*n*) of the diſtance $Px = \sqrt{(a^2 \pm 2by + y^2)}$, and $n - \frac{1}{2}$ be an integer affirmative number or 0, the fluents Y and V of both the fluxions can be found in finite algebraical terms of *y*; if $n - \frac{1}{2}$ be an integer negative number, both the fluents can be found in the above-

mentioned finite terms together with the arc of a circle, whoſe radius is $\sqrt{a^2-b^2}$ and tangent $y=\mp b$, unleſs $n-\frac{1}{2}=-1$, in which caſe the fluent Y involves that circular arc, and alſo the logarithm of $y^2\pm 2by+a^2$. If $n-\frac{1}{2}$ denotes a fraction whoſe denominator is 2, both the fluents can be expreſſed by the finite terms together with the log. of $y\pm b+\sqrt{(y^2\pm 2by+a^2)}$. If the fluents be given, when n is a given quantity, and $n-\frac{1}{2}$ not a whole affirmative number, from them can be deduced the fluents of any fluxions reſulting by increaſing or diminiſhing n by a whole number, unleſs in the above-mentioned caſe of $n-\frac{1}{2}=-1$. If $b=0$, and conſequently the line Pf is perpendicular to the given line ab, the fluent Y will be expreſſed by the finite terms, unleſs $n-\frac{1}{2}=-1$, in which caſe it will be as $\frac{1}{2}$ log. (y^2+a^2) when properly corrected.

Theſe fluxions $\dot{Y}$ and $\dot{V}$ may be transformed into others, whoſe variable quantity is P$x=u$ the diſtance from P, by ſubſtituting in the fluxions for y and $\dot{y}$ their reſpective values $\sqrt{(u^2-a^2+b^2)}\mp b$ and $\frac{u\dot{u}}{\sqrt{(u^2-a^2+b^2)}}$, and conſequently for $\sqrt{(y^2\pm 2by+a^2)}$ its value u.

PROBLEM II.

Fig. 3. Given the attraction of each of the parts of a given ſurface in terms of their diſtance from a given point P, and an equation expreſſing the relation between an abſciſs A$p=x$, and its correſpondent ordinates $pm=y$ of the ſurface; to find the attraction of the ſurface on the given point P.

Firſt, by the preceding propoſition find the attractions Y and V of any ordinate $m\,p\,m'$ in the directions of the ordinate pm and of the line Pp; and from the equation expreſſing the relation

relation between the abſciſs and ordinates of the given curve, find the abſciſs in terms of the ordinates $(pm)=\pi:(y)$, and thence $\dot{x}=\varphi:(y)\times\dot{y}$ and $\surd(a'^2\pm 2sa'x+x^2)=\varphi':(y)$, where $PA=a'$ and $s=$ coſine of the angle, which the abſciſs Ap makes with the line PA; then find the fluents of the three fluxions $\dot{x}\times Y=\dot{y}\times Y\times\varphi:(y)$, $\dot{x}\times\frac{V\times x}{\surd(a'^2\pm 2sa'x+x^2}=\varphi:(y)\times\dot{y}\times\frac{\pi:(y)}{\varphi:(y)}\times V$ and $\dot{x}\times\frac{a'V}{\surd(a'^2\pm 2sa'x+x^2)}=\dot{y}\times\frac{a'V}{\varphi':(y)}$ contained between the values of y, which correſpond to the extreme values of x, which ſuppoſe Y′, V′, and Z; and draw through the point P the lines Py and Pz reſpectively parallel to the ordinates pm and to the abſciſs Ap and equal to $r\times Y'$ and V′; aſſume Pu in the line $(PA)=t\times Z$, r and t denoting the ſines of the angles, which the ordinates pm and line AP make with the abſciſs Ap: reduce theſe three forces Py, Pz, and Pu, to one Pf, and Pf will be the force of the ſurface on the point P.

Cor. 1. If for y and $\dot{y}$ be ſubſtituted their values in terms of x and $\dot{x}$, deduced from the equation expreſſing the relation between the abſciſs Ap and ordinate pm of the given curve, thence will be deduced the above-mentioned fluents Y, V, Y′, V′, and Z, in terms of x; and in the ſame manner, if for x and $\dot{x}$ be ſubſtituted in the fluxions or fluents reſulting their values $\surd(u^2-a'^2+1^2a'^2)\mp sa'$, and its fluxion, there will reſult the above-mentioned fluxions or fluents in terms of u the diſtance from the point P.

Cor. 2. Let the curve be a circle, of which A is the center, PA a line perpendicular to the plane of the circle, and the ordinate pm perpendicular to the abſciſs Ap; the forces on each ſide of the abſciſs Ap will be equal, and the force in the direc-

tion of the abſciſs Ap will be equal to that in the contrary direction; the force in the direction (PA) $= 4 \times \int \frac{a\dot{u}}{\sqrt{(u^2-a^2)}} \times \int \frac{u\dot{y}}{\sqrt{u^2+y^2}} \times F : (\sqrt{(u^2+y^2)}) = W$, in which $F : (\sqrt{u^2+y^2})$ is the function of the diſtance, according to which the given force on the particles varies; the fluent $\int \frac{u\dot{y}}{\sqrt{(u^2+y^2)}} \times F : \sqrt{(u^2+y^2)}$ is contained between the values o and $\sqrt{(r^2+a^2-u^2)}$ of the quantity y, and the fluent W is contained between the values a and $\sqrt{(a^2+r^2)}$ of the quantity u, where $a = PA$ and r the radius of the circle; but the ſame force is $= 2 \times 3,14159$ &c. $\times \int a\dot{u} \times F : (u)$, where $F : (u)$ denotes the given function of the diſtance (u), and the fluent is contained between the values a and $\sqrt{a^2+r^2}$ of u.

PROBLEM III.

To find the attraction of a given ſolid on a given point P. Find the attraction of every parallel ſection on that point by the preceding problem, and multiply it into the correſpondent fluxion of the firſt abſciſſa AP, and alſo find the fluent of the reſulting fluxion, which, properly corrected, multiply into the ſine of the angle, which the firſt abſciſſa makes with the parallel ſections, and the product will be proportional to the attraction of the ſolid on the given point P.

2. Fig. 4. Let the ſolid ABCH be generated by the rotation of a given curve round its axis AB, which paſſes through the point attracted P, and this ſolid be ſuppoſed to conſiſt of ſmall evaneſcent ſolids, whoſe baſes are the ſurfaces EF, *ef*, &c. of ſpheres, of which the center is P, and altitudes F*f*, &c. the

increments of the baſe AB contained between the two contiguous ſurfaces EF and *ef*: from the points E and *e* of the curve draw ED and *ed* perpendicular to the axis AB, and ES perpendicular to the arc E*e* of the given curve at the point E, and meeting the axis AB in S; then will the evaneſcent ſolid $EF\mathit{fe} = p \times PE \times FD \times F\mathit{f} = p \times FD \times PS \times D\mathit{d}$ (becauſe $F\mathit{f} = \frac{PS \times D\mathit{d}}{PE}$) $= p \times \overline{\sqrt{(z^2+y^2)} - z} \times \overline{z\dot{z} \mp y\dot{y}}$, where z and y denote reſpectively the abſciſs PD, and its correſpondent ordinate DE of the given curve.

The increment of the attraction of the ſurface EF on the point P in the direction PD will be as the increment of the ſurface $(p \times PE \times D\mathit{d}) \times \frac{PD}{PE}$ × force of each particle $= p \times PD \times D\mathit{d} \times$ given force of the particle; but the fluent of the fluxion $PD \times D\mathit{d}$ contained between the points E and F is $= \frac{1}{2}PE^2 - \frac{1}{2}PD^2 = \frac{1}{2}ED^2$, whence the attraction of the evaneſcent ſolid $EF\mathit{fe}$ is as $\frac{1}{2}p \times ED^2 \times F\mathit{f} \times F : (\sqrt{x^2+y^2})$ force of each given particle at the diſtance $(PE = \sqrt{(x^2+y^2)}) = \frac{1}{2}p \times ED^2 \times \frac{PS}{PE} \times D\mathit{d} \times F : (\sqrt{z^2+y^2}) = \frac{1}{2}py^2 \times \frac{z\dot{z} \mp y\dot{y}}{\sqrt{(z^2+y^2)}} \times F : (\sqrt{(z^2+y^2)})$; the fluent of which, properly corrected, is as the attraction of the ſolid on the point P; p denotes the circumference of a circle, whoſe radius is 1.

Cor. 1. The fluxion of this ſolid is $\frac{1}{2}py^2\dot{z} = \dot{Y}$ which deduced from the preceding principles $= p \times (\sqrt{(z^2+y^2)} - z) \times (z\dot{z} \mp y\dot{y}) = \dot{V}$, and conſequently their fluents between two values of z, which correſpond to two values of $y = 0$, will be equal to each other.

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Cor. 2. The increment of the attraction of this ſolid as given in this propoſition $\frac{1}{2}p \times y^2 \times \frac{z\dot{z} \mp y\dot{y}}{\sqrt{(z^2+y^2)}} \times F : (\sqrt{(z^2+y^2)} = \dot{U}$, but in the preceding propoſition the force of a circle on the point $P = p \times \int a\dot{u} \times F : (u)$, where $u = \sqrt{(z^2+y^2)}$ and $a = z$, and y or u the only variable quantity contained in the fluxion; and conſequently the fluxion of the attraction of the ſolid $p \times \dot{z} \int z\frac{y\dot{y}}{\sqrt{z^2+y^2}} \times F : ((z^2+y^2)^{\frac{1}{2}}) = \dot{W}$; therefore, if for the fluent of $\frac{zy\dot{y}}{\sqrt{(z^2+y^2)}} \times F : ((z^2+y^2)^{\frac{1}{2}})$ be ſubſtituted its fluent contained between the values a and the value of y, which in the given equation correſponds to z; then the fluents of $\dot{U}$ and $\dot{W}$ contained between the two values of z, which correſponds to two values of $y = 0$, will be equal to each other.

The difference of the fluents of $\dot{Y}$ and $\dot{V}$, &c. contained between any other two values of z, can eaſily be deduced from the difference of two ſegments of ſpheres.

1. It may not be improper to remark in this place, that from different methods of finding the ſum of quantities, the fluents of fluxions, the integrals of increments, &c. quantities may often be deduced equal, which otherwiſe cannot without ſome difficulty; of which inſtances are contained in the Meditationes, and I ſhall here ſubjoin one or two more to thoſe already given in this Paper.

Ex. 1. Any curvilinear area ABC, &c. may be ſuppoſed to conſiſt of evaneſcent areas EF*ef*, of which the baſe EF is the arc of a circle, whoſe radius is $PE = \sqrt{(z^2+y^2)}$ and ſine $ED = y$, and altitude F*f*, and conſequently the fluxion of the area $= Ff \times$ arc (A) of a circle whoſe radius is PE and ſine $ED =$

$\frac{PS}{PE} \times \dot{z} \times A = \frac{z\dot{z} \mp y\dot{y}}{\sqrt{(z^2+y^2)}} \times A = \dot{V}$; the fluent of $\dot{V}$ contained between the two values of z which correſpond to two values of $(y)=0$ will be equal to the fluent of $y\dot{z}$ contained between the ſame two values of z.

Ex. 2. The attraction of any circular arc EF in the direction PD on a point P (P being the center of a circle, of which EF is an arc, and ED the ſine of that arc) will be as ED × force at diſtance $PE = ED \times F : (PE)$; for the attraction in the direction $PD/\!/F$ at the point x is as the increment of the arc $xy \times F : (PE) \times \frac{Pl}{Px}$ (xl and yl' being at right angles to PF) $= U \times \frac{Px}{xl} \times \frac{Pl}{Px} \times F : (PE) = \frac{U' \times Pl}{ix} \times F : (PE) = \frac{u\dot{u}}{\sqrt{PE^2-u^2}} \times F : (PE)$, if $u = Pl$; and conſequently the fluent of it is as $\sqrt{PE^2-u^2} \times F : (PE) = ED \times F : (PE)$, and the attraction of the ſurface $EFef$ will be as $ED \times Ff \times F : (PE) = ED \times \dot{z} \times \frac{PS}{PE} \times F : (PE) = y \times \frac{z\dot{z} \mp y\dot{y}}{\sqrt{(z^2+y^2)}} \times F : ((z^2+y^2)^{\frac{1}{2}} = \dot{V}$; the attraction of the curve will alſo vary as $\int \dot{z} \int \frac{zu \times F : ((u^2+z^2)^{\frac{1}{2}})}{(z^2+u^2)^{\frac{1}{2}}} = W$, in which the fluent of $\frac{zu \times F : ((z^2+u^2)^{\frac{1}{2}})}{(z^2+u^2)^{\frac{1}{2}}}$ is contained between $u = 0$ and $u = y$; the fluents of $\dot{V}$ and $\dot{W}$ contained between two values of z, which correſpond to two values of $y = 0$, will be equal to each other.

2. From a ſimilar method may be deduced equalities between other like fluents, for the curve may be ſuppoſed to conſiſt of other ſimilar curve ſurfaces equally as circles, and the ſolid of ſimilar ſegments of other ſolids equally as ſpheres.

3. From the ſame principles may innumerable ſerieſes equal to each other be deduced; for by different converging ſerieſes find

find the ſum of the ſame quantity or quantities, and there will reſult ſerieſes equal to each other: for inſtance (fig. 5), if the time of falling down the arcs AC and BC and their interpolations from the principles delivered in the Meditationes Analyticæ, of which the difference let be D; find the difference between the times of a body's falling through BC when it began to fall from A and from B by a ſeries proceeding according to the dimenſions of AB = o′ a ſmall quantity; and find, by a ſeries of the ſame kind, the time of falling through AB; the ſum of theſe two ſerieſes will be equal to D. Similar propoſitions may be deduced from fluxional equations.

4. In ſome caſes the ratios of the times of bodies falling through ſome particular diſtances to each other may be eaſily known; for inſtance, let the force vary as the $m-1$ power of the diſtance (x), and a be the diſtance from which the body began to fall, then the velocity varies as $\sqrt{(a^m - x^m)}$, and the increment of the time as $\frac{\dot{x}}{\sqrt{(a^m - x^m)}}$; but if the parts of different curves are proportional, then will a, x, and $\dot{x}$ vary in the ſame ratio as each other, and conſequently the time through proportional parts of the diſtance will vary as $a^{\frac{2-m}{2}}$; and if the bodies be reſiſted likewiſe by a force which varies as the $\frac{2m-2}{m}$ power of the velocities, then the times through proportional parts will vary as before, that is, as $a^{\frac{2-m}{2}}$, where a denotes the proportional diſtances from the points where the forces and reſiſtances are equal.

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PROBLEM IV.

1. Fig. 6. Given an equation expreſſing the relation between the two abſciſſæ $z = AP$ and $x = Pp$ and their correſpondent ordinates $y = pm$ of a ſolid, to find its ſolid contents contained between two values of its firſt abſciſſæ z. Aſſume z as an invariable quantity, and from the equation reſulting find the fluent Z of $y\dot{x}$ contained between the extreme values of x or y; then find the fluent of $Z\dot{z}$ contained between the given values of z, and the fluent multiplied into the product of the ſines of the angles, which the firſt abſciſſa makes with the plane of the ordinates and ſecond abſciſs, and the ſecond abſciſs makes with its correſpondent ordinates, will be the ſolid content required.

2. Fig. 7. Let the firſt abſciſs z of a ſolid be perpendicular to the planes of the ordinates, and the ſecond abſciſs $Pp = x$ perpendicular to the ordinates themſelves $pm = y$. Firſt, aſſume the firſt abſciſs as invariable, and find the increment of the arc $p'm = (\dot{x}^2 + \dot{y}^2)^{\frac{1}{2}}$, then aſſume the ſecond abſciſs Pp as conſtant, and let mu be the fluxion of the ordinate y or u, when the fluxion of the firſt abſciſs is $\dot{z} = ul$, where ul is perpendicular to the plane of the ordinates $p'pm$, and l a point of the ſurface of the ſolid; draw uh perpendicular to the arc $p'm$, and ſince ul is conſtituted at right angles to the plane $pp'm$, lh will cut the arc $p'm$ at right angles; but $uh = \frac{um \times pp'}{p'm} = \frac{u\dot{x}}{\sqrt{(\dot{x} + \dot{y}^2)}}$, $lh = (hu^2 + lu^2)^{\frac{1}{2}} = \left(\frac{u\dot{x}}{\dot{x}^2 + \dot{y}^2} + \dot{z}^2\right)^{\frac{1}{2}}$ the fluxion of the ſurface will be $lh \times \sqrt{(\dot{x}^2 + \dot{y}^2)}$. From the given equation expreſſing the relation between the two abſciſſæ z and x and ordinates y find, by aſſuming z invariable $p\dot{x} = \dot{y}$, and by aſſuming x invariable $q\dot{z} = \dot{y}' = \dot{u}$,

$\dot{y}' = \dot{u}$, which being ſubſtituted for their values in the quantity $lh \times \sqrt{(\dot{x}^2 + \dot{y}^2)}$, there will reſult $(q^2 + p^2 + 1)^{\frac{1}{2}} \times \dot{x} \times \dot{z} = A\dot{x}\dot{z} = \frac{(q^2 + p^2 + 1)^{\frac{1}{2}}}{p} \times \dot{y} \times \dot{z} = B\dot{y}\dot{z}$; in A and B for y and x reſpectively ſubſtitute their value deduced from the given equation, and let the reſulting quantities be $A'\dot{x}\dot{z}$ and $B'\dot{y}\dot{z}$, where A' is a function of x and z, and B' a function of y and z; find the fluent of $A'\dot{x}\dot{z}$ from the ſuppoſition that x is only variable contained between the extreme values of x to a given value of z, which let be $L\dot{z}$, then find the fluent of $L\dot{z}$ by ſuppoſing z only variable contained between given values of z, and it will be the ſurface of the ſolid contained between thoſe values.

The ſame may be deduced by finding the fluent of $B'\dot{y}\dot{z}$ on the ſuppoſition that y is the only variable quantity contained between the extreme values of y as before of x to a given value of z, which let be $L'\dot{z}$; then will the fluent of $L'\dot{z}$ contained between the given values of z be the ſurface required.

If the ſolid be a cone generated by the rotation of a rectangular triangle round a ſide containing the right angle as an axis; hu will be a given quantity, if $\dot{z}$ be given.

If the above-mentioned angles are given, but not right ones, the arc $p'm$ and perpendicular $l'h$ can eaſily be deduced, and conſequently the increment of the ſurface.

3. To define a curve of double curvature, it is neceſſary to have two equations expreſſing the relation between the abſciſſæ z and x and their ordinates (y) given, and if the angles which they reſpectively make with each other be right ones; the fluxion of the arc as given in the *Proprietates Curvarum* is $(\dot{z}^2 + \dot{x}^2 + \dot{y}^2)^{\frac{1}{2}}$. Find its value from the two given equations in terms of x, y, or z, multiplied into its reſpective fluxions, and its fluent, properly corrected, will be the length of the arc required.

If

If the angles are not right, they may eaſily be reduced to them.

4. The attractions of theſe ſurfaces, curves, &c. on a given point P may be deduced from the preceding principles of finding the attractions of each of the parts in the directions of the firſt abſciſſa, which paſſes through the point P, the ſecond abſciſſa, and the ordinates, and then finding the integrals of theſe increments.

From the method which determines the attraction of a body, ſurface, &c. on a given point can be determined the attraction of a body, &c. on any number of points, and conſequently the attraction of one body, &c. on another, &c.

It is ſometimes advantageous to transform the firſt abſciſs, that it may paſs through the point attracted; and the abſciſſæ and ordinates, that they may be at right angles to each other, &c.

PROBLEM V.

1. Fig. 8. Given an equation expreſſing the relation between the two abſciſſæ AP and Pp of a ſolid, and their correſpondent ordinates pm, or AP′, P′p', and $p'm'$; to transform the firſt abſciſſa into any other Lh.

Let the abſciſſa Lh begin from a point L of the firſt abſciſſa AP, and meet an ordinate pm in the point h; draw hp, and let the ſines of the angles Ppm, Php, and pPh; LPh, PhL, and PLh, be denoted reſpectively by r, s, and t, &c. r', s', and t'; through a point h of the line Ph draw $p'h'm'$ parallel to pm, and L$h = z$, $hh' = x$, and $h'm' = y$: in the given equation for AP, Pp, and pm ſubſtitute reſpectively their correſpondent

reſpondent values $\frac{s'z}{r'} \pm AL\,(a)$, $\frac{st'z}{rr'} \pm \frac{sx}{r}$ (for $Ph = \frac{t'z}{r'}$ and $Ph' = Ph \pm hh' = \frac{t'z}{r'} \pm x$), and $y \pm \frac{tt'z}{rr'} \pm \frac{tx}{r'}$; there reſults an equation to the ſame ſolid expreſſing the relation between the two abſciſſæ $z = Lh$ and x, and their correſpondent ordinates y.

1. 2. If the abſciſs Lh does not begin from L, a point in the firſt given abſciſs AP, but from M a point given out of it, it may be reduced to the preceding caſe, by drawing from M a line $MN = c$ to the plane of the firſt and ſecond abſciſſæ parallel to the ordinates pm; and from N to the firſt abſciſſa a line $NO = b$ parallel to the ſecond abſciſſæ, and ſubſtituting in the equation expreſſing the relation between AP, Pp, and pm for AP, Pp, and pm reſpectively $z \pm AO$ (a), $x \pm b$ and $y \pm c$; and there reſults the equation required expreſſing the relation between the two abſciſſæ z and x, and their correſpondent ordinates y, of which the firſt abſciſſa z paſſes through the point M.

2. To change the ſecond abſciſſa Pp into any other Lh, the firſt abſciſſa and ordinates remaining the ſame. In the preceding figure let L be conſidered as a moveable point of the firſt abſciſs AL, and the ſines of the reſpective angles denoted by the ſame letters as before, and $Lh = x$, $AL = z$, and $hm = y$; in the given equation for AP, Pp, and pm, ſubſtitute $z \pm \frac{s'x}{r'}$, $\frac{st'x}{rr'}$, and $y \pm \frac{tt'x}{rr'}$; and there will reſult the equation required expreſſing the relation between z and x the abſciſſæ, and their correſpondent ordinates y.

3. Fig 8. To change the ordinates, the abſciſſæ remaining the ſame, draw $p'm$ an ordinate transformed, $p'h$ parallel to the firſt abſciſſa AP, and meeting a ſecond abſciſſa, of which

pm is an ordinate in h: for the ſines of the angles $p'hp$, hpp', and $hp'p$; $p'pm$, pmp', and $pp'm$ write r, s, and t, r', s', and t'; and for AP′, P′p', and $p'm$ reſpectively z, x, and y; then ſubſtitute in the given equation for AP, Pp, and pm, their reſpective values $z\,(\mathrm{AP}') = \pm\frac{ss'}{rr'} \times y$, $x\,(\mathrm{P}'p' = \pm\frac{ts'}{rr'}y$, and $\frac{t'}{r'}y$; and there reſults an equation to the ſolid expreſſing the relation between the two abſciſſæ AP′ and P′p' and the tranformed ordinates $p'm$.

From theſe caſes, which are eaſily reducible to one, may be transformed any given abſciſſæ and their correſpondent ordinates into any other containing given angles, &c. with the before-mentioned abſciſſæ and ordinates.

In the properties of curve lines, firſt publiſhed in 1762, is given a method of deducing the equation to any ſection of the ſolid, and in particular the caſe of deducing the equation to the projection of any curve on a given plane.

From the principles given in this, and the Paper on centripetal forces, which the Royal Society did me the honour to print, can be deduced the fluxional equations, whoſe fluents expreſs the relations between the abſciſſæ and their correſpondent ordinates of the curves deſcribed by bodies, of which the particles act on each other with forces varying according to given functions of their diſtances.

Fig. 1.

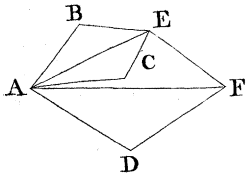


Fig. 2.

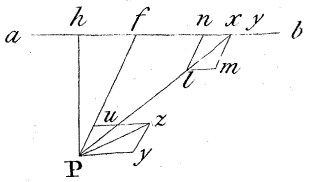
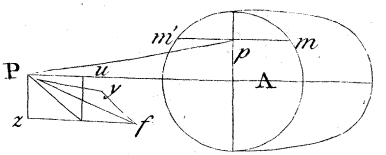


Fig. 3.



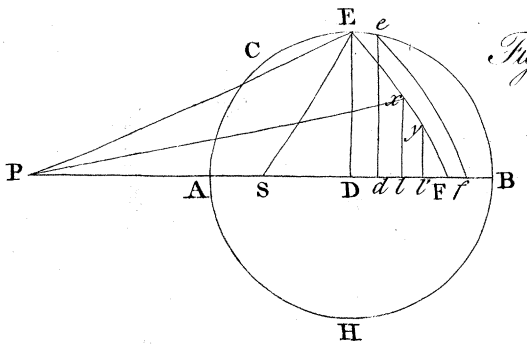
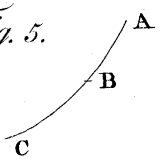


Fig. 4.

Fig. 5.



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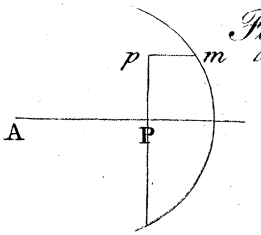

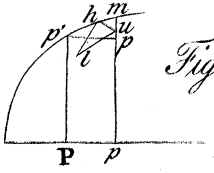

Fig. 7.

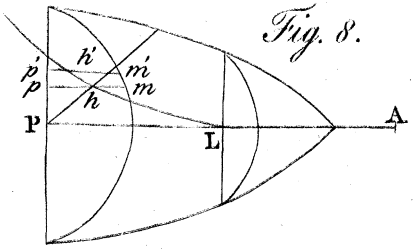

Fig. 8.

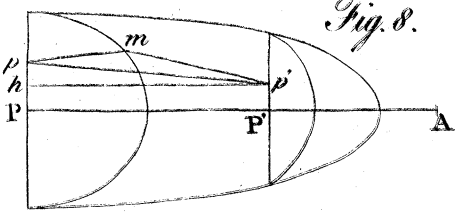

Fig. 8.